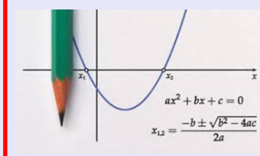
Math 125

Spring 2022

Lecture 17



Simplify:

1) $\sqrt[2]{16}=4$

check: $4^2=16$

2) $-\sqrt{\frac{4}{25}}=-\frac{2}{5}$

Check: $-\left(\frac{2}{5}\right)^2=-\frac{4}{25}$

3) $\sqrt{4^2+3^2}=\sqrt{16+9}=\sqrt{25}$

$=\boxed{5}$

4) $\sqrt[2]{-100}$ $\to -100<0$ Undefined

index=2
even index

Radicand ≥ 0

5) $\sqrt{10^2-6^2}$

$=\sqrt{100-36}$

$=\sqrt{64}=\boxed{8}$

Check $8^2=64$

$\text{Answer}^{\text{Index}}=\text{Radicand}$

Find the distance between A(-4, 1) and B(2, 9).

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-4-2)^2 + (1-9)^2} = \sqrt{(-6)^2 + (-8)^2}$$

$$= \sqrt{36 + 64} = \sqrt{100} = \boxed{10}$$

Find the domain for $f(x) = \sqrt{6-2x}$ in interval notation.

No index → index = 2 → even index

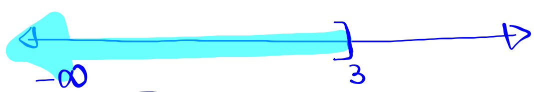
Radicand ≥ 0

$$6 - 2x \geq 0$$

$$-2x \geq -6$$

$$\frac{-2}{-2}x \leq \frac{-6}{-2}$$

$$\boxed{x \leq 3}$$



$$\boxed{(-\infty, 3]}$$

Simplify

1) $\sqrt[3]{8} = \boxed{2}$ ✓

$2^3 = 8$ ✓

2) $\sqrt[3]{-27} = \boxed{-3}$

$(-3)^3 = -27$ ✓

3) $\sqrt[3]{\frac{1}{125}} = \frac{\sqrt[3]{1}}{\sqrt[3]{125}} = \boxed{\frac{1}{5}}$

4) $\sqrt[3]{-64} = \boxed{-4}$

$\text{Answer}^{\text{Index}} = \text{Radicand}$

$(-4)^3 = -64$

Even index:
Radiand ≥ 0, Answer ≥ 0

odd index:
Radicand & Answer must have Same Sign.
both +, or both −.

Simplify:

1) $\sqrt[4]{1} = \boxed{1}$

2) $\sqrt[5]{-1} = \boxed{-1}$

odd

3) $\sqrt[6]{-1}$ Undefined

even index

Radicand ≥ 0

Radicand < 0

4) $\sqrt[7]{1} = \boxed{1}$

So $\sqrt[n]{1} = 1$

5) $\sqrt[4]{16} = \boxed{2}$

6) $\sqrt[5]{-32} = \boxed{-2}$

7) $\sqrt[6]{-64}$ undefined

even index

Radicand < 0

8) $\sqrt[6]{64} = \boxed{2}$

$(2)^6 = 64$

Radical rules:

Assume $x \geq 0$

$\sqrt[n]{x^n} = \boxed{x}$

$\sqrt[4]{x^4} = x$, $\sqrt[7]{x^7} = x$, $\sqrt[5]{x^5} = x$

$\sqrt[3]{(-3)^3} = -3$ $\quad$ $\sqrt{(-5)^2} = |-5| = 5$

final Ans.

More Radical Rules:

$$\sqrt[n]{AB} = \sqrt[n]{A}\,\sqrt[n]{B}$$

$$\sqrt[n]{\frac{A}{B}} = \frac{\sqrt[n]{A}}{\sqrt[n]{B}}$$

Simplify

$$\sqrt[3]{40} = \sqrt[3]{8\cdot 5}$$

$$= \sqrt[3]{8}\cdot\sqrt[3]{5}$$

$$= \boxed{2\sqrt[3]{5}}$$

Simplify $\sqrt{40} = \sqrt{4\cdot 10}$

$$= \sqrt{4}\sqrt{10} = \boxed{2\sqrt{10}}$$

Simplify

1) $\sqrt{50} = \sqrt{25\cdot 2}$

$50\cdot 1$

$25\cdot 2$

$10\cdot 5$

$$= \sqrt{25}\sqrt{2}$$

$$= \boxed{5\sqrt{2}}$$

2) $\sqrt[3]{54} = \sqrt[3]{27\cdot 2}$

$1\cdot 54$

$2\cdot 27$

$3\cdot 18$

$6\cdot 9$

$$= \sqrt[3]{27}\,\sqrt[3]{2}$$

$$= \boxed{3\sqrt[3]{2}}$$

Simplify $\sqrt{\frac{49}{64}} = \frac{\sqrt{49}}{\sqrt{64}} = \boxed{\frac{7}{8}}$

Simplify $\sqrt{10}\cdot\sqrt{8} = \sqrt{10\cdot 8}$

$$= \sqrt{80} = \sqrt{16\cdot 5}$$

$$= \sqrt{16}\sqrt{5}$$

$$= \boxed{4\sqrt{5}}$$

Simplify $\sqrt{6}\cdot\sqrt{12}$

$$= \sqrt{6\cdot 12} = \sqrt{72} = \sqrt{36\cdot 2} = \sqrt{36}\sqrt{2}$$

$$= \boxed{6\sqrt{2}}$$

Distribute and Simplify

$$\sqrt{5}\left(\sqrt{10}+\sqrt{5}\right)$$

$$=\sqrt{5}\sqrt{10}+\sqrt{5}\sqrt{5}$$

$$=\sqrt{50}+\sqrt{25}$$

$$=\sqrt{25}\sqrt{2}+\sqrt{25}=\boxed{5\sqrt{2}+5}$$

Distribute and Simplify:

$$\sqrt{3}\left(2\sqrt{6}-\sqrt{30}\right)=2\sqrt{18}-\sqrt{90}$$

$$=2\sqrt{9}\sqrt{2}-\sqrt{9}\sqrt{10}$$

$$=2\cdot 3\sqrt{2}-3\sqrt{10}$$

$$=\boxed{6\sqrt{2}-3\sqrt{10}}$$

FOIL and Simplify:

$$\left(\sqrt{6}+2\right)\left(\sqrt{6}+5\right)$$

$$=\sqrt{36}+5\sqrt{6}+2\sqrt{6}+10$$

$$=6+7\sqrt{6}+10=\boxed{16+7\sqrt{6}}$$

FOIL and Simplify:

$$\left(3\sqrt{2}+4\right)\left(3\sqrt{2}-4\right)$$

$$=9\sqrt{4}\cancel{-12\sqrt{2}}\cancel{+12\sqrt{2}}-16$$

$$=9\cdot 2-16$$

$$=18-16=\boxed{2}$$

Simplify $(\sqrt{5}-\sqrt{3})^2$

Hint: $x^2 = x \cdot x$

$$(\sqrt{5}-\sqrt{3})^2 = (\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})$$

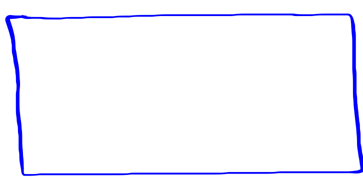
FOIL & Simplify

$$= \sqrt{25} - \sqrt{15} - \sqrt{15} + \sqrt{9}$$

$$= 5 - 2\sqrt{15} + 3$$

$$= \boxed{8 - 2\sqrt{15}}$$

Consider the rectangle below:



$2\sqrt{5} - 2$

$3\sqrt{5} + 2$

$$P = 2L + 2W$$

$$= 2(3\sqrt{5} + 2) + 2(2\sqrt{5} - 2)$$

$$= 6\sqrt{5} \cancel{+4} + 4\sqrt{5} \cancel{-4}$$

$$= \boxed{10\sqrt{5}}$$

$$A = LW$$

$$= (3\sqrt{5} + 2)(2\sqrt{5} - 2)$$

$$= 6\sqrt{25} - 6\sqrt{5} + 4\sqrt{5} - 4$$

$$= 6 \cdot 5 - 2\sqrt{5} - 4$$

$$= 30 - 2\sqrt{5} - 4 = \boxed{26 - 2\sqrt{5}}$$

Solve by Subs. method:

$$\begin{cases} x - y = -1 \\ y = x^2 + 1 \end{cases}$$

$x - (x^2 + 1) = -1$

$x - x^2 \cancel{-1} = \cancel{-1}$

$x - x^2 = 0$

$x(1 - x) = 0$

$x = 0$ $\quad$ $1 - x = 0$

$x = 1$

If $x = 0$

$y = 0^2 + 1 = 1 \Rightarrow (0,1)$

If $x = 1$

$y = 1^2 + 1 = 2 \Rightarrow (1,2)$

Final Ans: $\{(0,1), (1,2)\}$

Solve by addition Method:

$$\begin{cases} 16x^2 - 4y^2 = 72 \\ x^2 - y^2 = 3 \end{cases}$$

Notice:
First equation can be reduced if we divide by 4

$$\begin{cases} 4x^2 - y^2 = 18 \\ x^2 - y^2 = 3 \end{cases} \cdot (-1) \Rightarrow \begin{cases} 4x^2 - y^2 = 18 \\ -x^2 + y^2 = -3 \end{cases}$$

$3x^2 = 15$

$x^2 = 5$ $\quad$ $\boxed{x = \pm\sqrt{5}}$

$5 - y^2 = 3$

$y^2 = 2 \Rightarrow \boxed{y = \pm\sqrt{2}}$

Final Ans: $\{(\sqrt{5}, \sqrt{2}), (\sqrt{5}, -\sqrt{2}), (-\sqrt{5}, \sqrt{2}), (-\sqrt{5}, -\sqrt{2})\}$

Solve

$$\begin{cases} x - 3y = -5 \\ x^2 + y^2 = 25 \end{cases}$$

Hint: Isolate one Variable, then use Subs. Method

$x = 3y - 5$

$(3y-5)^2 + y^2 = 25$

$(3y-5)(3y-5) + y^2 = 25$

$9y^2 - 15y - 15y \cancel{+25} + y^2 = \cancel{25}$

$10y^2 - 30y = 0$

Divide by 10

$y^2 - 3y = 0$

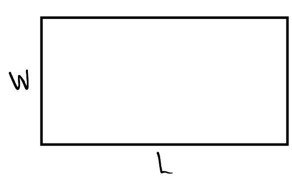
$y(y-3) = 0$

$\boxed{y=0}$ $x = 3(0) - 5)$ $\boxed{x=-5}$

$\boxed{y=3}$ $x = 3(3) - 5$ $\boxed{x=4}$

$\{(-5,0),(4,3)\}$

Find the dimensions of a rectangle with perimeter 40 feet and area 96 square feet.



$$\div 2\begin{cases} 2L + 2W = 40 \\ LW = 96 \end{cases}$$

$L = 20 - W$

$$\begin{cases} L + W = 20 \\ LW = 96 \end{cases}$$

$(20-W)W = 96$

$20W - W^2 = 96$

$W^2 - 20W + 96 = 0$

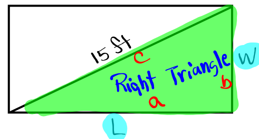
$(W-8)(W-12) = 0$

$W-8=0$	$W-12=0$
$W=8$	$W=12$
$L=12$	$L=8$

Final Ans:

8 ft by 12 ft

Find L & W is Area is 108 ft².



$a^2+b^2=c^2$

$LW=108$

Pythagorean Thrm

$L^2+W^2=15^2$

$\begin{cases} LW=108 \\ L^2+W^2=225 \end{cases} \longrightarrow W=\frac{108}{L}$

$L^2+\left(\frac{108}{L}\right)^2=225$ $\quad L^2+\frac{11664}{L^2}=225$

multiply by L^2 to clear fraction.

$L^2\cdot L^2+\cancel{L^2}\cdot\frac{11664}{\cancel{L^2}}=L^2\cdot 225$

$L^4+11664=225L^2$

$L^4-225L^2+11664=0$

$(L^2-144)(L^2-81)=0$

$L^2=144$ $\quad L^2=81$

$L=12$ $\quad L=9$

$LW=108$

$12\ W=108$ $\quad W=9$

$9\ W=108$ $\quad W=12$

Dimensions are

9 ft by 12 ft

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y varies directly as x^4.

y is 128 when x is 2.

Find y when x is 5.

Exam 2: April 26th.

$y=Kx^4$

$128=K\cdot 2^4$

$128=16K$

$K=\frac{128}{16}$ $\quad \boxed{K=8}$

$y=8x^4$

$y=8(5)^4$

$\boxed{y=5000}$

y varies inversely as cube root of x.

y is 10 when x is 1000.

Find y when x is 125.

$$y = \frac{K}{\sqrt[3]{x}}$$

$$10 = \frac{K}{\sqrt[3]{1000}}$$

$$10 = \frac{K}{10}$$

$$\boxed{K = 100}$$

$$y = \frac{100}{\sqrt[3]{x}}$$

$$y = \frac{100}{\sqrt[3]{125}} = \frac{100}{5} = 20$$

$$\boxed{y = 20}$$

Class QZ 15:

Solve

$$\begin{cases} x^2 + y^2 = 13 \\ x^2 - y^2 = 5 \end{cases}$$

$$2x^2 = 18$$

$$x^2 = 9$$

$$\boxed{x = \pm 3}$$

$$9 + y^2 = 13$$

$$y^2 = 4$$

$$\boxed{y = \pm 2}$$

Final Ans: $\boxed{\{(3,2),(3,-2),(-3,2),(-3,-2)\}}$